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II. Solution by the Proposer.

It is convenient to use the notation

$$\Delta = \begin{vmatrix} x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \\ x_1^{n-2} & x_2^{n-2} & \cdots & x_n^{n-2} \\ \cdot & \cdot & \cdots & \cdot \\ x_1 & x_2 & \cdots & x_n \\ 1 & 1 & \cdots & 1 \end{vmatrix} = \prod_{h,j=1}^{n} (x_h - x_j), \qquad h < j.^1$$

It is also convenient to designate the minor of Δ with respect to x_i^{n-1} by Δ_i.

$$\Delta_i = \prod_{h,j=1}^{n}{}' (x_h - x_j), \qquad h < j,$$

where the prime indicates that no factor containing x_i enters. It now follows at once that

$$\sum_{i=1}^{n} \frac{x_i^{n-1}}{\prod'_{h=1}^{n} (x_i - x_h)} = \sum_{i=1}^{n} \frac{x_i^{n-1}\Delta_i}{(-1)^{i-1} \prod_{h=1}^{i-1} (x_h - x_i) \prod_{h=i+1}^{n} (x_i - x_h) \prod'_{h,j=1}^{n} (x_h - x_j)}$$

$$= \sum_{i=1}^{n} \frac{(-1)^{i-1} x_i^{n-1}\Delta_i}{\prod_{h,j=1}^{n} (x_h - x_j),\ h < j} = \frac{1}{\Delta} \sum_{i=1}^{n} (-1)^{i-1} x_i^{n-1} \Delta_i = \frac{1}{\Delta}\Delta = 1. \qquad h < j,$$

Also solved by A. A. Bennett, G. W. Hartwell, and E. H. Worthington.

GEOMETRY.

508. Proposed by J. E. ROWE, State College, Pa.

The trilinear coördinates of the Brocard triangle are $(s_3^3, s_1s_2s_3, s_1^3)$, $(s_2^3, s_1^3, s_1s_2s_3)$, and $(s_1s_2s_3, s_0^3, s_2^3)$ where s_i $(i = 1, 2, 3)$ are the sines of the angles of the fundamental triangle. Show that the Brocard triangle and the fundamental triangle are in perspective, and that the trilinear coördinates of the center of perspectivity are s_i^{-3} $(i = 1, 2, 3)$ instead of s_i^3 which are incorrectly given in Clebsch *Vorlesungen über Geometrie*, p. 323.

Solution by William Hoover, Columbus, Ohio.

Use of homogeneous coördinates permits us to replace s_1, s_2, s_3 by a, b, c and we have

$$(0, 2\Delta/b, 0),\ (c^3, abc, a^3); \qquad (0, 0, 2\Delta/c),\ (b^3, a^3, abc); \qquad (2\Delta/a, 0, 0),\ (abc, c^3, b^3)$$

for the pairs of sets of trilinear coördinates of the corresponding vertices of the fundamental and Brocard triangles.

The equation of a straight line passing through two points $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$ is

$$\alpha(\beta_1\gamma_2 - \beta_2\gamma_1) + \beta(\gamma_1\alpha_2 - \gamma_2\alpha_1) + \gamma(\alpha_1\beta_2 - \alpha_2\beta_1) = 0. \tag{1}$$

Thus the straight lines joining the pairs of vertices are

$$a^3\alpha - c^3\gamma = 0 \quad (2), \qquad -a^3\alpha + b^3\beta = 0 \quad (3), \qquad b^3\beta - c^3\gamma = 0 \quad (4).$$

The three lines

$$l_1\alpha + m_1\beta + n_1\gamma = 0 \quad (5), \qquad l_2\alpha + m_2\beta + n_2\gamma = 0 \quad (6), \qquad l_3\alpha + m_3\beta + n_3\gamma = 0 \quad (7)$$

are concurrent if

$$\begin{vmatrix} l_1, & m_1, & n_1 \\ l_2, & m_2, & n_2 \\ l_3, & m_3, & n_3 \end{vmatrix} = 0. \tag{8}$$

(2), (3), and (4) are, therefore, concurrent and the fundamental and Brocard triangles are in perspective.

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[1] L. G. Weld, *The Theory of Determinants*, Art. 89, p. 169.

The proportional values of α, β, γ in (5) and (6) are given by

$$\frac{\alpha'}{m_1n_2 - m_2n_1} = \frac{\beta'}{n_1l_2 - n_2l_1} = \frac{\gamma'}{l_1m_2 - l_2m_1} \tag{9}$$

and those in (2) and (3) by

$$\frac{\alpha'}{b^3c^3} = \frac{\beta'}{a^3c^3} = \frac{\gamma'}{a^3b^3} \quad (10), \qquad \text{or} \qquad \alpha' \Big/ \frac{1}{a^3} = \beta' \Big/ \frac{1}{b^3} = \gamma' \Big/ \frac{1}{c^3} \quad (11),$$

satisfying (4) and proving the theorem.

The fundamental triangle is in perspective with two other Brocard triangles, the centers of perspective being $1/b^2$, $1/c^2$, $1/a^2$; $1/c^2$, $1/a^2$, $1/b^2$.

Also solved by J. W. Clawson, C. P. Sousley, and S. W. Reaves.

509. Proposed by NORMAN ANNING, Chilliwack, B. C.

A picture whose coördinates are (0, 0), (50, 0), (50, 50), and (0, 50) is repeated on a smaller scale as part of itself with the coördinates (7, 0), (31, 7), (24, 31), (0, 24). Locate the vanishing point.

Solution by S. W. Reaves, University of Oklahoma.

Let P, P_1, P_2, $\cdots$ denote the picture and its successive images, and let O_1, O_2, O_3, $\cdots$ be the successive images of the first vertex. We shall find the vanishing point by determining the limiting position of O_n as n increases indefinitely.

It readily follows from the data of the problem that the side of P_1 is one half that of P and makes with it an angle $\theta = \tan^{-1} 7/24$. It is clear that the side of P_n bears the same relations to the side of P_{n-1} for all values of n.

By projecting the successive segments OO_1, O_1O_2, O_2O_3, $\cdots$ on the coördinate axes, it readily follows that the coördinates of the limiting position of O_n are given by the following infinite series:

$$x = 7 + \frac{7}{2}\cos\theta + \frac{7}{2^2}\cos 2\theta + \cdots + \frac{7}{2^n}\cos n\theta + \cdots,$$

$$y = \frac{7}{2}\sin\theta + \frac{7}{2^2}\sin 2\theta + \cdots + \frac{7}{2^n}\sin n\theta + \cdots.$$

Multiplying the second equation through by $i = \sqrt{-1}$, adding the result to the first equation, denoting $\cos\theta + i\sin\theta$ by v, and remembering that $\cos n\theta + i \sin n\theta$ is by De Moivre's formula equal to v^n, we have

$$x + iy = 7\left[1 + \frac{v}{2} + \left(\frac{v}{2}\right)^2 + \cdots + \left(\frac{v}{2}\right)^n + \cdots\right].$$

Using the formula for the sum of a geometric progression and then replacing v by its value

$$\frac{24}{25} + i\frac{7}{25},$$

we have

$$x + iy = \frac{364}{29} + i\frac{98}{29}.$$

Hence, the coördinates of the required vanishing point are (364/29, 98/29).

Also solved by J. B. Reynolds and W. R. Ransom.

CALCULUS.

424. Proposed by OSCAR S. ADAMS, Washington, D. C.

What is the value of

$$\frac{\Gamma'(1)}{\Gamma(1)} - \frac{\Gamma'(\frac{1}{2})}{\Gamma(\frac{1}{2})}.$$